The optimal control approach to analyze some inverse problems for reaction-diffusion systems arising from epidemiology

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In this talk, we consider the parameter identification problem for two kind of reaction-diffusion systems: the compartmental model susceptible-infected–susceptible (SIS) with diffusion. The direct problem that we consider is a susceptible-infected-susceptible mathematical model with cross-diffusion, which was deduced by assuming the following hypotheses: The total population can be partitioned into susceptible and infected individuals; a healthy susceptible individual becomes infected through contact with an infected individual; there is no immunity, and infected individuals can become susceptible again; the spread of epidemics arises in a spatially heterogeneous environment; the susceptible and infected individuals implement strategies to avoid each other by staying away. The spread of the dynamics is governed by an initial boundary value problem for a reaction-diffusion system, where the model unknowns are the densities of susceptible and infected individuals, and the boundary condition models the fact that there is neither emigration nor immigration through their boundary. The reaction consists of two terms: modeling disease transmission and infection recovery and the diffusion cross-diffusion matrix arising from the assumption that the motion of susceptibles is affected by taxis. The inverse problem is the determination of rates on the reaction and diffusion terms from observation of an infected profile on a fixed time. We reformulated the identification problem as an optimal control problem where the cost function is a regularized least squares function. The fundamental contributions of this article are the following: The existence of at least one solution to the optimization problem or, equivalently, the diffusion identification problem; the introduction of first-order necessary optimality conditions; and the necessary conditions that imply a local uniqueness result of the inverse problem. Moreover, we present some numerical results and the extension of the methodology to reaction-diffusion systems arising from oncology.

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References

[1] N. C. Apreutesei, *A optimal control problem for a pest , predator, and plant system*. Nonlinear Anal. Real World Appl. 13 (2012) 1391–1400.

[2] A. Coronel, L. Friz, I. Hess, M. Zegarra, *On the existence and uniqueness of an inverse problem in epidemiology*. Applicable Analysis, 100(3)(2021), 513–526

[3] A. Coronel, F. Huancas, I. Hess, A. Tello, *The diffusion identification in a SIS reaction-diffusion system.* Mathematical Biosciences and Engineering. 21 (2024) 562–581.

[4] A. Coronel, F. Huancas, E. Lozada, M. Rojas-Medar, *The Dubovitskii and Milyutin methodology applied to an optimal control problem originating in an ecological system*. Mathematics, 9 (2021) 479.

[5] A. Coronel, F. Huancas, E. Lozada, M. Rojas-Medar, *Results for a control problem for a SIS epidemic reaction–diffusion model*, Symmetry, 15(6) (2023) 1224.

[6] A. Coronel, F. Huancas, M. Sepúlveda, *A note on the existence and stability of an inverse problem for a SIS model*, Comp. and Math. with App. 77(2019) 3186–3194.

[7] A. Coronel, F. Huancas, M. Sepúlveda, *Identification of space distributed coefficients in an indirectly transmitted diseases model*. Inverse Probl. 11 (2019) 115001.

[8] F. Guillén-González, E. Mallea, M. Rodrgüez, *A regularity criterion for a 3D chemo-repulsion system and its application to a bilinear optimal control problem*, SIAM J. Control Optim. 58 (3) (2020) 1457–1490.

[9] Y. Tao, M. Winkler, *Analysis of a chemotaxis-SIS epidemic model with unbounded infection force.* Nonlinear Analysis: Real World Applications, 71 (2023) 103820.

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