Boundary null controllability of a class of 2-d degenerate parabolic PDEs

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In this talk we deal with the boundary null controllability of some degenerate parabolic equations posed on a square domain, presenting, as far as we know, the first study of boundary controllability for such equations in multidimensional settings. The proof combines two classical techniques: the method of moments and a Lebeau-Robbiano strategy. A key novelty of this work lies in the analysis of boundary control localized on a subset of the boundary where degeneracy occurs. Let us consider the following degenerate parabolic equation in a square domain $\Omega = (0,1) \times (0,1)$. We study the degenerate parabolic equation:

$$(1) \quad \begin{cases} \partial_t u = \operatorname{div}(D\nabla u) & \text{in } (0,T) \times \Omega, \\ u(t) = 1_\gamma q(t), & \text{in } (0,T) \times \partial\Omega, \\ u(0) = u_0, & \text{in } \Omega. \end{cases}$$

where $\gamma = \{0\} \times \omega$ with ω an open subset of $(0,1)$ (in the y variable). The matrix function $D : \overline{\Omega} \mapsto M_{2\times 2}(\mathbb{R})$ is given by

$$D(x,y) = \begin{pmatrix} x^{\alpha_1} & 0 \\ 0 & y^{\alpha_2} \end{pmatrix},$$

where $\alpha = (\alpha_1, \alpha_2) \in [0,1) \times [0,1)$, and u_0 is the initial data that lies in a functional space $H_\alpha^{-1}(\Omega)$ We will also give results for $\alpha_1, \alpha_2 \in (1,2)$ and combination of different grades of degeneracy (weak/strong, strong-strong, etc.) [2] We use the ideas in [1] to obtain the boundary control.

Acknowledgements

Bibliography

[1] Assia Benabdallah, Franck Boyer, Manuel González-Burgos, and Guillaume Olive. Sharp estimates of the one-dimensional boundary control cost for parabolic systems and application to the N-dimensional boundary null controllability in cylindrical domains. *SIAM J. Control Optim.*, 52(5):2970–3001, 2014.

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