On the null-controllability of subelliptic systems of Grushin type

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In this talk, we will discuss the null-controllability properties of the heat equation associated with Grushin-type operators, posed on tensorized domains of two dimensions. That is, parabolic equations associated with second-order operators whose coefficients can be either singular and/or degenerated. Our parabolic operator of interest takes the form

$$(1) \qquad \partial_t - \partial_x^2 - q(x)^2 \partial_y(r(y)^2 \partial_y) + V(x)$$

where, q a non-zero function that can vanish, r a strictly positive function, and V is a potential that depends on the choice of a measure on the domain, eventually singular.

We will first discuss negative results, that may be obtained via Agmon theory. Then, we will talk about positive results, either using a constructive approach, as the moments method, or by means of Carleman estimates.

Finally, if time lets us, we shall briefly discuss how these results extend on two-dimensional almost-Riemannian manifolds, for which the metric is degenerated along a submanifold of codimension one.

References

[1] R. Vanlaere,, *Non Null-Controllability Properties of the Grushin-Like Heat Equation on 2D-Manifolds*. Preprint, 2025. https://hal.science/hal-04940947

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