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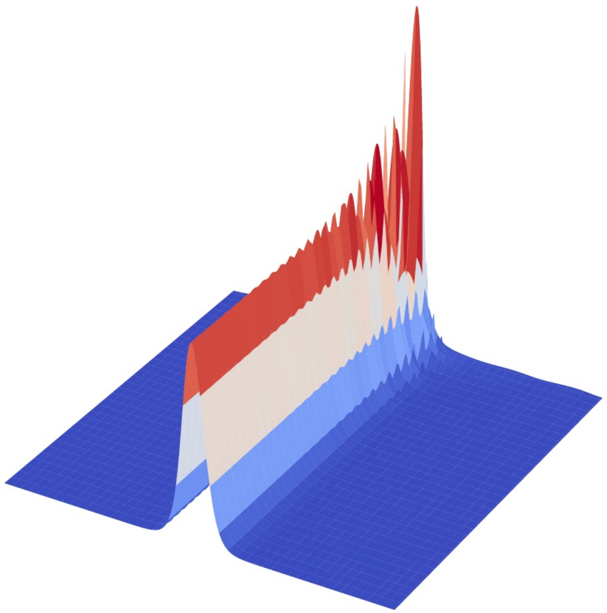
STABILITY OF NONLINEAR DIRAC SOLITONS UNDER THE ACTION OF EXTERNAL POTENTIALS



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Stability of nonlinear Dirac solitons under the action of external potential

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ABSTRACT

The instabilities observed in direct numerical simulations of the Gross–Neveu equation under linear and harmonic potentials are studied. The Lakoba algorithm, based on the method of characteristics, is performed to numerically obtain the two spinor components. We identify non-conservation of energy and charge in simulations with instabilities, and we find that all studied solitons are numerically stable, except the low-frequency solitons oscillating in the harmonic potential over long periods of time. These instabilities, as in the case of the Gross–Neveu equation without potential, can be removed by imposing absorbing boundary conditions. The dynamics of the soliton is in perfect agreement with the prediction obtained using an *Ansatz* with only two collective coordinates, namely, the position and momentum of the center of mass. We employ the temporal variation of both field energy and momentum to determine the evolution equations satisfied by the collective coordinates. By applying the same methodology, we also demonstrate the spurious character of the reported instabilities in the Alexeeva–Barashenkov–Saxena model under external potentials.

The nonlinear Dirac equation in 1 + 1-dimensions supports localized solitons. Theoretically, these traveling waves propagate with constant velocity, energy, momentum, and charge. However, the soliton profiles can be distorted, and eventually destroyed, due to intrinsic and/or numerical instabilities. The constants of motion and the initial profiles can also be modified by external potentials, which may give rise, in some cases, to instabilities. The perturbations can render the soliton unstable (intrinsic instability). The difference between the initial condition and the exact solution of the perturbed nonlinear Dirac equation, the boundary conditions, and the relationship between the space and time steps, among other reasons, may create numerical instabilities (spurious instabilities). Lakoba's numerical algorithm shows that solitons of the Gross–Neveu equation (nonlinear Dirac equation with scalar–scalar self-interaction) are stable regardless of their frequency, whereas, by using either operator splitting or Runge–Kutta methods, instabilities for low frequencies are observed (due to the interaction of the soliton with its own dispersive radiation). In this work, Lakoba's method of characteristics is employed to numerically solve the massive Gross–Neveu and the Alexeeva–Barashenkov–Saxena models under spatial potentials. It is shown that, in both systems, the initial soliton remains stable, in most of its domains of existence. This enables it to be concluded that the reported instabilities in the aforementioned two systems were numerical and that their solitary waves are stable within the range of parameters studied.

I. INTRODUCTION

The spectral stability analysis of nonlinear waves starts with the linearized equations describing small deviations around the static or stationary nonlinear wave. The stability of the waves is determined by the sign of the real part of the eigenvalues of the corresponding Sturm–Liouville problem, and the predicted instability can be observed numerically. This is the standard procedure employed to demonstrate the stability (or instability) of the kinks in nonlinear Klein–Gordon equations,[1–3] of the enveloped solitons in the

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NON-LINEAR DIRAC EQUATION IN (1+1) DIMENSIONS

GROSS-NEVEU (GN) MODEL

$$i\gamma^\mu\partial_\mu\Psi - \Psi + (\bar{\Psi}\Psi)\Psi - e\gamma^\mu A_\mu\Psi = 0$$

$$\Psi(x,t) = \{\psi(x,t);\chi(x,t)\}^T,$$

ALEXEEVA-BARASHENKOV-SAXENA (ABS) MODEL

$$i\gamma^\mu\partial_\mu\Psi + \Psi + (\bar{\Psi}\Psi)\Psi - \frac{1}{2}(\bar{\Psi}\gamma_\mu\Psi)\gamma^\mu\Psi - e\gamma^\mu A_\mu\Psi = 0$$

$$\Psi(x,t) = \{v(x,t);u(x,t)\}^T$$

$$\bar{\Psi} = \Psi^\dagger\gamma^0, \quad A_1 = 0, \quad eA_0 = V(x), \quad \gamma^0 = \sigma_3, \quad \gamma^1 = i\sigma_2, \quad \gamma_0 = \gamma^0, \quad \gamma_1 = -\gamma^1$$

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- **Theoretically** → conservation of both charge and energy

Charge: $Q = \int_{-\infty}^{+\infty} dx\, \rho(x,t)$ with $\rho(x,t) = \bar{\Psi}\gamma^0\Psi$ **charge density**

Energy: $E_{GN} = \int_{-\infty}^{+\infty} dx T^{00}[\psi,\chi], \quad E_{ABS} = \int_{-\infty}^{+\infty} dx T^{00}[v,u]$ **energy density**

$$T^{00}[\Psi,\chi] = \frac{i}{2}\left[\chi\partial_x\psi^* + \psi\partial_x\chi^* - \chi^*\partial_x\psi - \psi^*\partial_x\chi\right] + V(x)(|\psi|^2 + |\chi|^2) + (|\psi|^2 - |\chi|^2) - \frac{1}{2}(|\psi|^2 - |\chi|^2)^2$$

$$T^{00}[v,u] = \frac{i}{2}\left[u\partial_x u^* + v^*\partial_x v - u^*\partial_x u - v\partial_x v^*\right] + V(x)(|u|^2 + |v|^2) - (uv^* + u^*v) - \frac{1}{2}\left[(uv^*)^2 + (u^*v)^2\right]$$

NON-LINEAR DIRAC EQUATION IN (1+1) DIMENSIONS

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$$i\gamma^\mu \partial_\mu \Psi - \Psi + (\bar{\Psi}\Psi)\Psi - e\gamma^\mu A_\mu \Psi = 0$$

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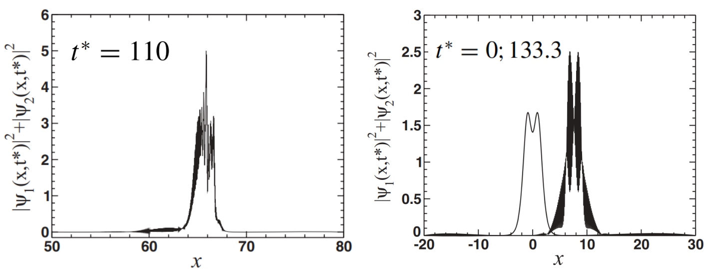
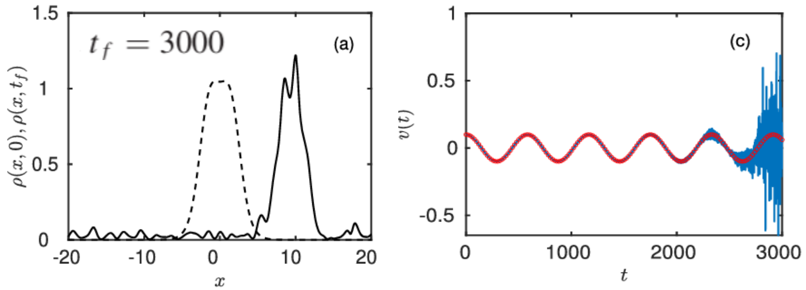
$$\Psi(x,t) = \{v(x,t); u(x,t)\}^T$$

$$\bar{\Psi} = \Psi^\dagger \gamma^0, \quad A_1 = 0, \quad eA_0 = V(x), \quad \gamma^0 = \sigma_3, \quad \gamma^1 = i\sigma_2, \quad \gamma_0 = \gamma^0, \quad \gamma_1 = -\gamma^1$$

- **Theoretically** → conservation of both charge and energy
- **Numerically** → instabilities are observed (RK methods, boundary conditions, numerical steps, etc.)



Phys. Rev. E 86, 046602 (2012)



J. Phys. A: Math. Theor. 54, 405702 (2021)

NUMERICAL ALGORITHM

GN model $(\kappa = m = 1)$
$u_{GN} = (\psi + \chi)/\sqrt{2},\, v_{GN} = (\psi - \chi)/\sqrt{2}$

ABS model $(\kappa = 0, m = -1)$

$$\begin{cases} \dfrac{\partial u}{\partial t} + \dfrac{\partial u}{\partial x} = i\,[\kappa\, u\, v^{\star} + u^{\star}\, v - m]v - i\,V(x)\,u \\ \dfrac{\partial v}{\partial t} - \dfrac{\partial v}{\partial x} = i\,[u\, v^{\star} + \kappa\, u^{\star}\, v - m]u - i\,V(x)\,v \end{cases}$$

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Characteristic coordinates
$\xi = \frac{t-x}{2}, \ \eta = \frac{t+x}{2}$

$$\begin{cases} \dfrac{\partial u}{\partial \eta} = i\,[\kappa\, u\, v^\star + u^\star\, v - m]v - i\,V(\eta - \xi)\,u = F(\kappa, \eta, \xi, u, v) \\ \dfrac{\partial v}{\partial \xi} = i\,[u\, v^\star + \kappa\, u^\star\, v - m]u - i\,V(\eta - \xi)\,v = F(\kappa, \eta, \xi, v, u) \end{cases}$$

NUMERICAL ALGORITHM

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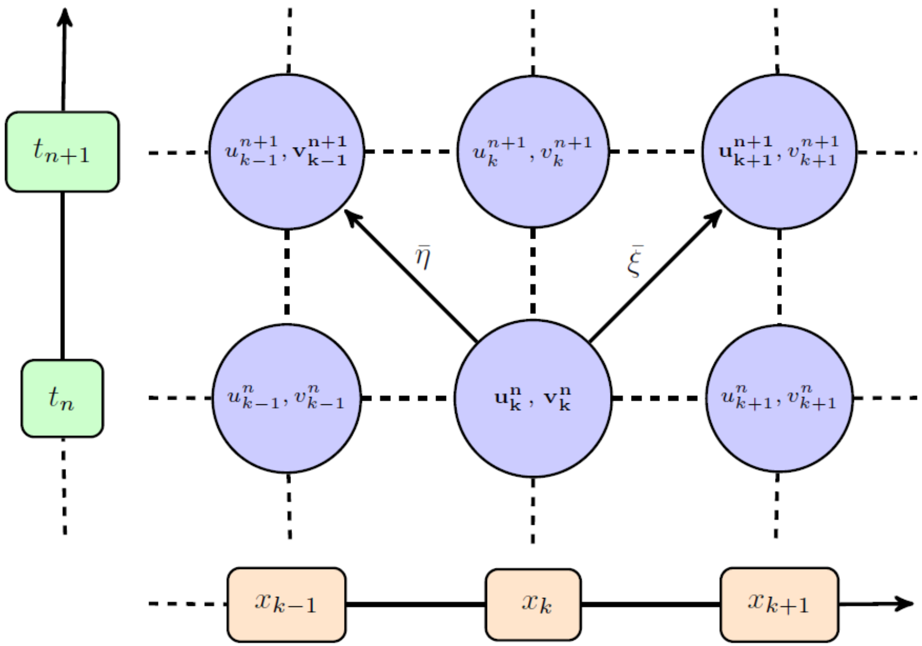
Characteristic coordinates

$\xi = \frac{t-x}{2},\ \eta = \frac{t+x}{2}$

$$\begin{cases} \dfrac{\partial u}{\partial \eta} = i\,[\kappa\, u\, v^\star + u^\star\, v - m]v - i\,V(\eta - \xi)\,u = F(\kappa, \eta, \xi, u, v) \\ \dfrac{\partial v}{\partial \xi} = i\,[u\, v^\star + \kappa\, u^\star\, v - m]u - i\,V(\eta - \xi)\,v = F(\kappa, \eta, \xi, v, u) \end{cases}$$

- **Spatial discretization** $(\Delta x = h)$: $x_k = -L + (k-1)h$ for $k = 1, 2, \dots, K$, $x_k \in [-L, L], K = \frac{2L}{h} + 1$
- **Temporal discretization** $(\Delta t = h)$: $t_n = (n-1)h$ for $n = 1, 2, \dots, N$, $t_n \in [0, t_f]$, $N = \frac{t_f}{h} + 1$
- **Characteristic coordinates discretization:** $(x_k, t_n) \rightarrow \left(\xi_k^n = \frac{t_n - x_k}{2},\ \eta_k^n = \frac{t_n + x_k}{2}\right)$

NUMERICAL ALGORITHM: PREDICTOR-CORRECTOR



Initial conditions:

u_k^1 and $v_k^1 \rightarrow$ exact moving at $t = 0$ (*Ansatz*)

Nonreflecting boundary conditions:

$$u_1^n = u_K^n = v_1^n = v_K^n = 0$$

- Predicted solution by **Simple Euler method**:

$$\bar{u}_{k+1}^{n+1} = u_k^n + hF\big(\kappa, \eta_k^n, \bar{\xi}, u_k^n, v_k^n\big) \quad \text{with } \bar{\xi} = \xi_k^n = \text{cte}$$

$$\bar{v}_{k-1}^{n+1} = v_k^n + hF\big(\kappa, \bar{\eta}, \xi_k^n, v_k^n, u_k^n\big) \quad \text{with } \bar{\eta} = \eta_k^n = \text{cte}$$

- Corrected solution by **trapezoidal rules**:

$$u_{k+1}^{n+1} = u_k^n + \frac{h}{2}\big[F\big(\kappa, \eta_k^n, \bar{\xi}, u_k^n, v_k^n\big) + F\big(\kappa, \eta_{k+1}^{n+1}, \bar{\xi}, \bar{u}_{k+1}^{n+1}, \bar{v}_{k+1}^{n+1}\big)\big]$$

$$v_{k-1}^{n+1} = v_k^n + \frac{h}{2}\big[F\big(\kappa, \bar{\eta}, \xi_k^n, v_k^n, u_k^n\big) + F\big(\kappa, \bar{\eta}, \xi_{k-1}^{n+1}, \bar{v}_{k-1}^{n+1}, \bar{u}_{k-1}^{n+1}\big)\big]$$

Phys. Lett. A 382, 300 (2018)

COLLECTIVE COORDINATE THEORY

$q(t)$ and $p(t)$ are the Collective Coordinates (CC)

$$\cosh\alpha(t) = \gamma(t) = 1/\sqrt{1-\dot{q}^2(t)}$$

$$\phi(z,t) = p(t)[z/\gamma(t) + q(t)] - \omega\gamma(t)t \qquad z(t) = \gamma(t)[x - q(t)] \qquad \beta = \sqrt{1-\omega^2}$$

- **Ansatz for GN model** $(0 < \omega < 1)$**:**

$$\tilde{\psi}(z,t) = \left(\cosh\frac{\alpha(t)}{2}A(z) + i\sinh\frac{\alpha(t)}{2}B(z)\right)e^{i\,\phi(z,t)}$$
$$\tilde{\chi}(z,t) = \left(\sinh\frac{\alpha(t)}{2}A(z) + i\cosh\frac{\alpha(t)}{2}B(z)\right)e^{i\,\phi(z,t)}$$

$$A(z) = \sqrt{2}\beta\frac{\sqrt{1+\omega}\cosh(\beta z)}{1+\omega\cosh(2\beta z)}$$

$$B(z) = \sqrt{2}\beta\frac{\sqrt{1-\omega}\sinh(\beta z)}{1+\omega\cosh(2\beta z)}$$

Slight modification of the exact solution for V(x)=0

- **Ansatz for ABS model** $(1/\sqrt{2} < \omega < 1)$**:**

$$\tilde{u}(z,t) = -e^{\alpha/2}\,a(z)\,e^{-i\,\theta(z)}\,e^{i\,\phi(z,t)}$$
$$\tilde{v}(z,t) = e^{-\alpha/2}\,a(z)\,e^{i\,\theta(z)}\,e^{i\,\phi(z,t)}$$

$$a^2(z) = \frac{[2(1-\omega)\mathrm{sech}^2(\beta z)][1+\lambda^2\tanh^2(\beta z)]}{1-6\lambda^2\tanh^2(\beta z)+\lambda^4\tanh^4(\beta z)}$$

$$\theta(z) = -\arctan[\lambda\tanh(\beta z)]$$

$$\lambda = \sqrt{\frac{1-\omega}{1+\omega}}$$

COLLECTIVE COORDINATE THEORY

- By inserting ansatz in T^{00} and by operating: $U(q,\dot{q}) = \int_{-\infty}^{+\infty} dz \, \frac{\rho(z,t)}{\gamma(t)} \, V\left(\frac{z}{\gamma} + q(t)\right)$ **particle potential**

$$p(t) = \omega \, \dot{q} \, \gamma - \frac{1}{Q} \frac{\partial U}{\partial \dot{q}}$$

$$\frac{d}{dt}[M_0 \, \gamma(t) \, \dot{q}(t)] = \frac{d}{dt} \frac{\partial U}{\partial \dot{q}} - \frac{\partial U}{\partial q}$$ **2nd Newton's law for center of mass**

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2nd Newton's law for center of mass

LINEAR POTENTIAL

$$V(x) = -V_1 \, x \;\longrightarrow\; U(q) = -V_1 \, Q \, q$$

$$M_0 = M_0(\omega)$$

***U* independent of $\dot q$!**

$$\begin{cases} p(t) = \omega \, \gamma \, \dot q \\ q(t) = q(0) + \dfrac{\sqrt{M_0^2 + (V_1 \, Q \, t)^2} - M_0}{V_1 \, Q} \end{cases}$$

One CC

COLLECTIVE COORDINATE THEORY

- By inserting ansatz in T^{00} and by operating: $U(q,\dot q) = \int_{-\infty}^{+\infty} dz \, \frac{\rho(z,t)}{\gamma(t)} V\left(\frac{z}{\gamma} + q(t)\right)$ **particle potential**

$$p(t) = \omega \, \dot q \, \gamma - \frac{1}{Q} \frac{\partial U}{\partial \dot q}$$

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2nd Newton's law for center of mass

LINEAR POTENTIAL

$$V(x) = -V_1 \, x \rightarrow U(q) = -V_1 \, Q \, q \quad \begin{cases} p(t) = \omega \, \gamma \, \dot q \\ q(t) = q(0) + \dfrac{\sqrt{M_0^2 + (V_1 \, Q \, t)^2} - M_0}{V_1 \, Q} \end{cases}$$

One CC

$M_0 = M_0(\omega)$

***U* independent of $\dot q$!**

HARMONIC POTENTIAL

Two CCs

$$V(x) = \frac{V_2}{2} x^2 \rightarrow U(q,\dot q) = \frac{V_2}{2}\left[q^2 \, Q + Q_2 \left(1 - \dot q^2\right)\right] \quad \begin{cases} p(t) = \dot q(t) \left(\omega \, \gamma(t) + V_2 \dfrac{Q_2}{Q}\right) \\ \left(M_0 \, \gamma^3(t) + V_2 \, Q_2\right) \ddot q(t) + V_2 \, Q \, q(t) = 0 \end{cases}$$

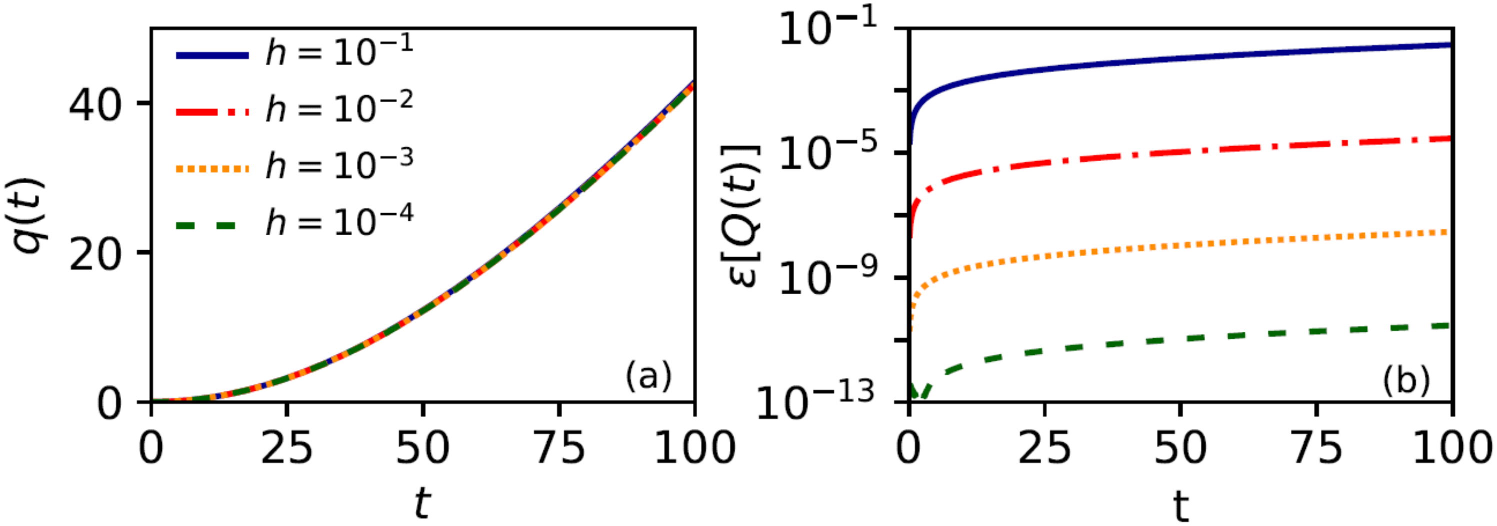
$(V_2 > 0)$

$Q_2 = \int_{-\infty}^{+\infty} dz \, z^2 \left[A(z)^2 + B(z)^2\right]$

Non-relativistic limit $(\dot q \ll 1, \gamma \approx 1) \rightarrow$ **simple pendulum eq.**

GN MODEL: LINEAR POTENTIAL $V(x) = -V_1 x$

$V_1 = 10^{-2}$, $L = 100$, $\omega = 0.9$, and $q(0) = \dot{q}(0) = p(0) = 0$



Position: $q(t) = \dfrac{1}{Q} \int_{-\infty}^{+\infty} dx\, \rho(x,t)\, x$

Relative error: $\epsilon[\mathcal{A}(t)] = \dfrac{|\mathcal{A}(t,h) - \mathcal{A}_e(t)|}{\mathcal{A}_e(t)}$ $\quad \mathcal{A}_e \equiv$ exact magnitude

Linear potential: $p(t) = \omega\, \dot{q}\, \gamma$ and $q(t) = q(0) + \dfrac{\sqrt{M_0^2 + (V_1\, Q\, t)^2} - M_0}{V_1\, Q}$ (one CC)

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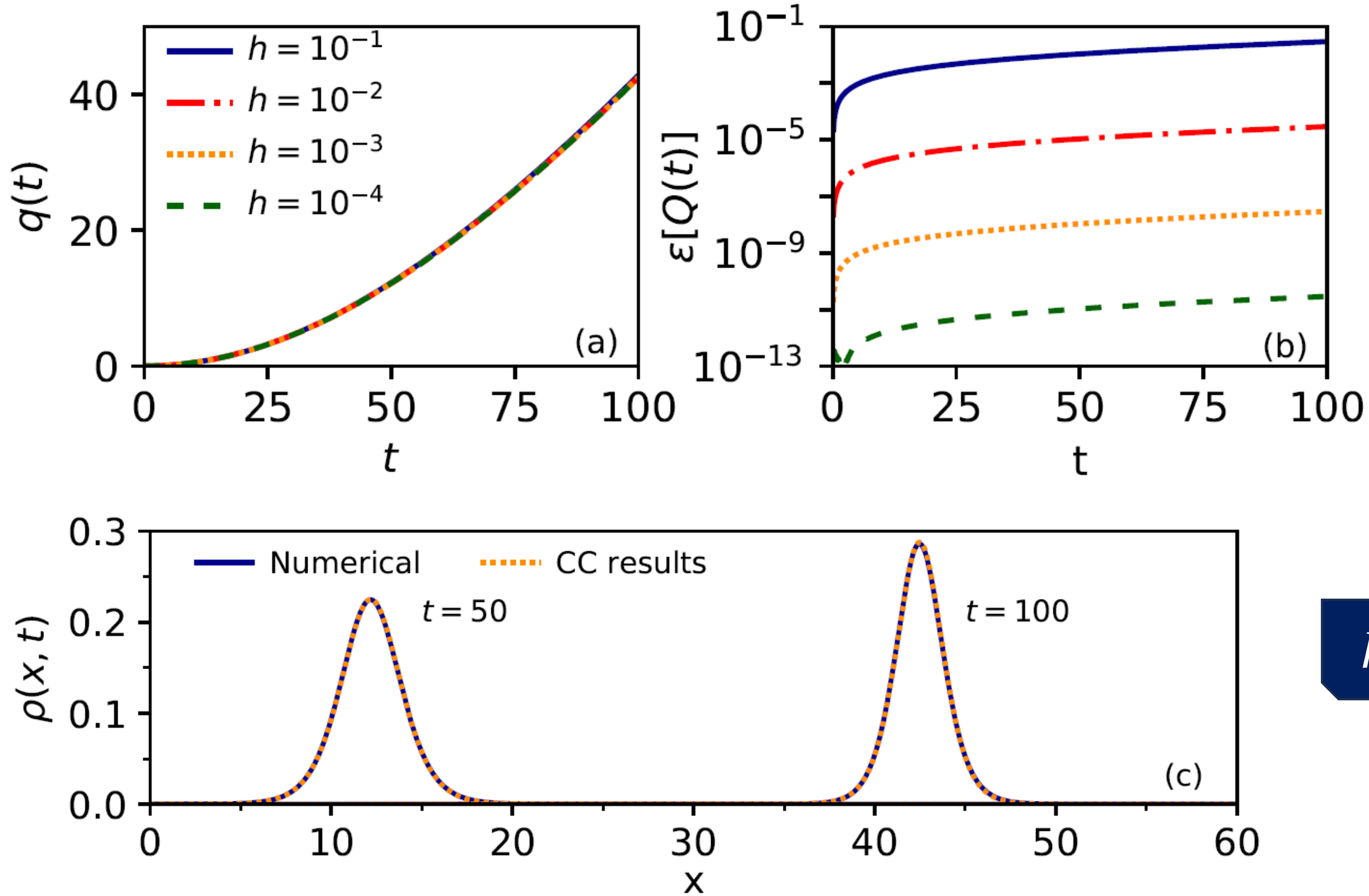
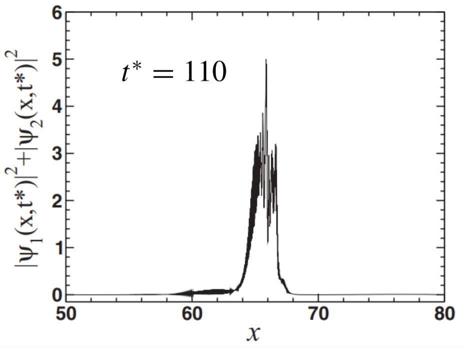


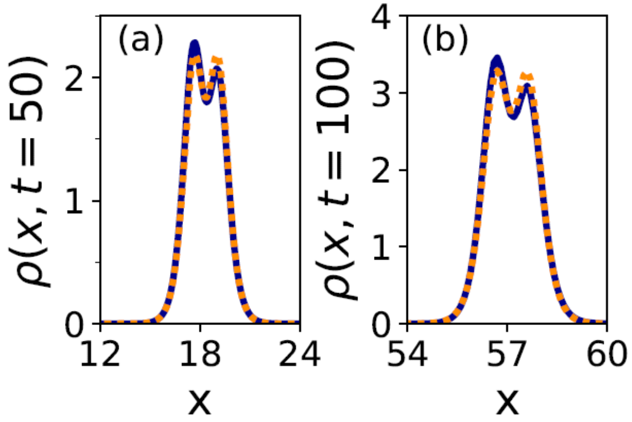
$h = 10^{-3}$

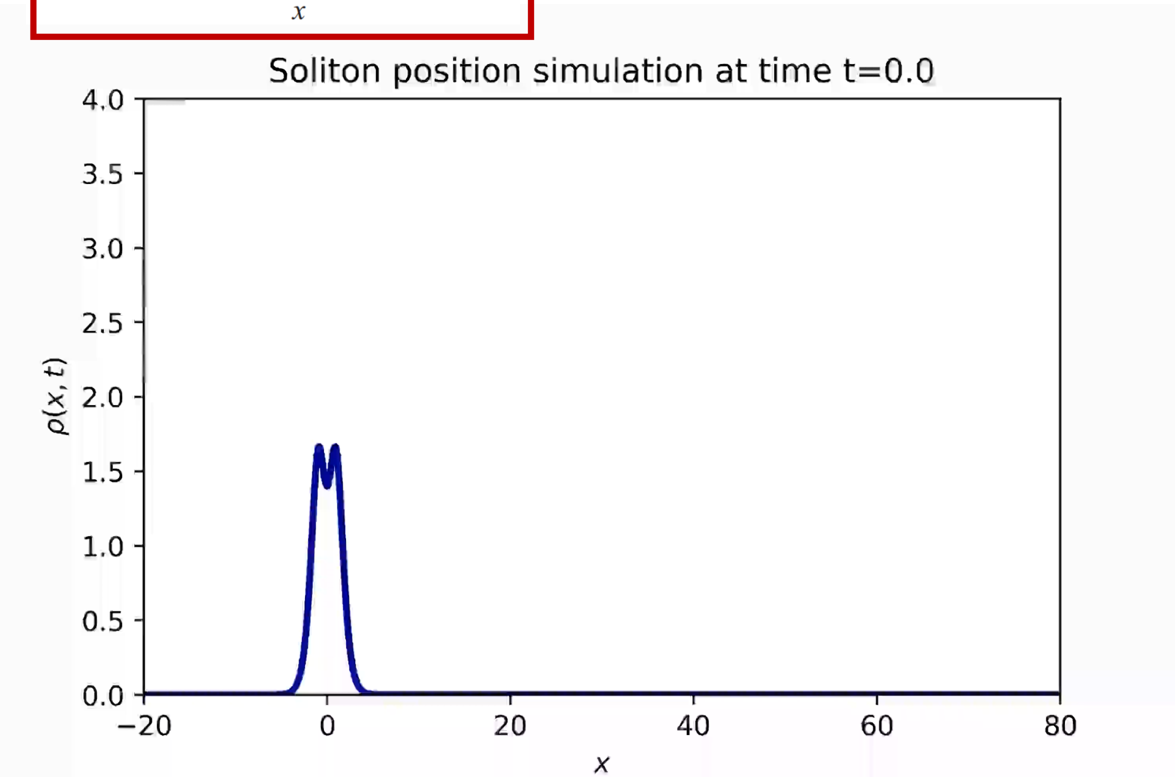
GN MODEL: LINEAR POTENTIAL $V(x) = -V_1 x$

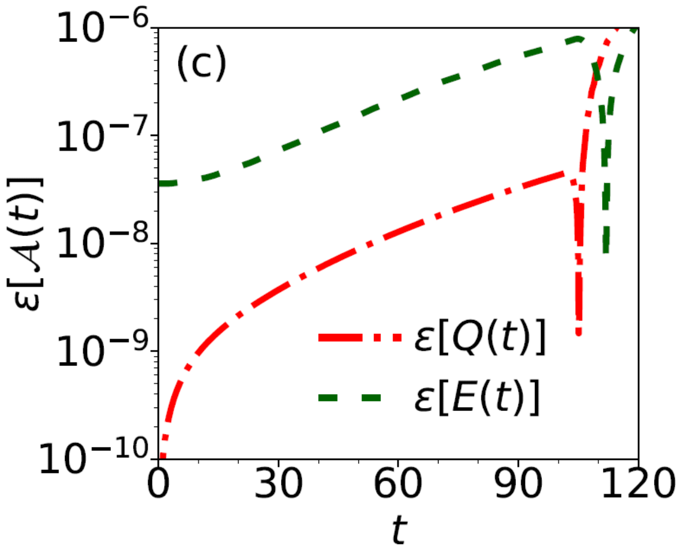
Phys. Rev. E 86, 046602 (2012)



$V_1 = 10^{-2}, L = 100$
$\omega = 0.3$ and
$q(0) = \dot{q}(0) = p(0) = 0$







GN MODEL: HARMONIC POTENTIAL $V(x) = \frac{1}{2} V_2 x^2$

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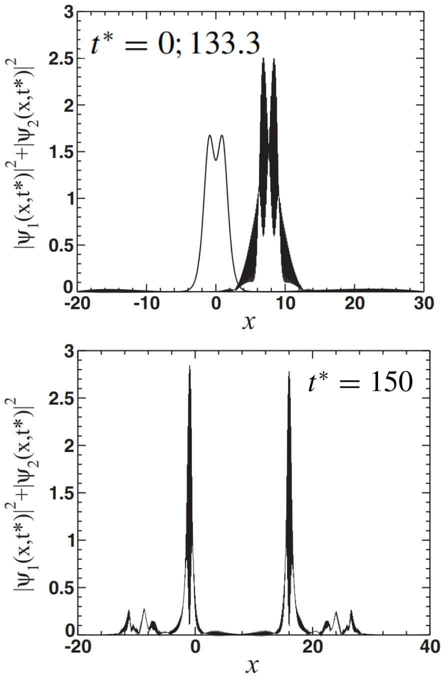
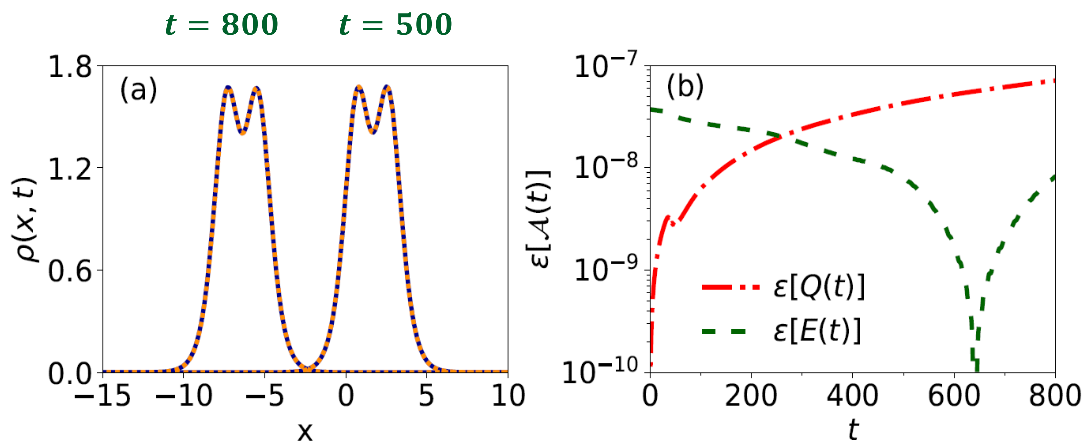
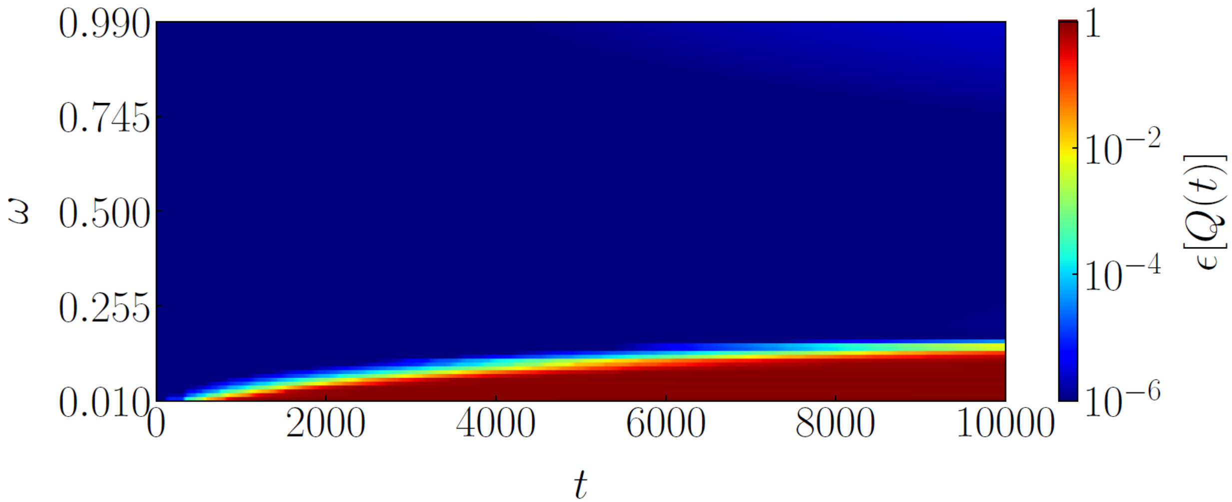


$V_2 = 10^{-4}, L = 100,$
$q(0) = 0, \dot{q}(0) = 0.1$
and $p(0) = p(\omega)$

$V_2 = 10^{-4}, L = 30,\ q(0) = 0,\ \dot{q}(0) = 0.1, \omega = 0.3, p(0) = p(\omega)$





GN MODEL: HARMONIC POTENTIAL $V(x) = \frac{1}{2}V_2 x^2$

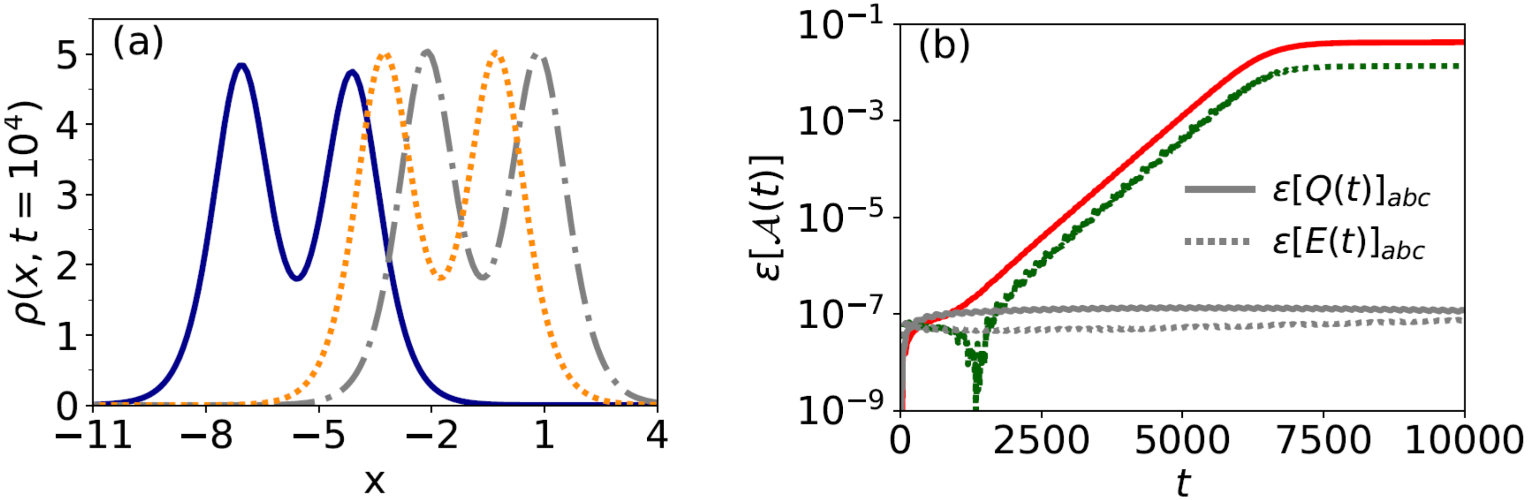
Absorbing boundary conditions (abc)

each temporal step h_{abc} the solution is multiplied by the function

$$\rho_a(x) = \begin{cases} e^{-\left(\frac{|x|-L_1}{W}\right)^2} & \text{if } |x| \in [L_1, L] \\ 1 & \text{if } |x| < L_1 \end{cases}$$

where $W = (L - L_1)/B$, $L_1 < L$ and B are parameters.

$V_2 = 10^{-4}$, $L = 30$, $q(0) = 0$, $\dot{q}(0) = 0.1$, $p(0) = 0.01$, $\omega = 0.1$
$L_1 = 0.4 \cdot L$, $B = 0.05$ and $h_{abc} = 0.4$



ABS MODEL: HARMONIC POTENTIAL $V(x) = \frac{1}{2}V_2 x^2$

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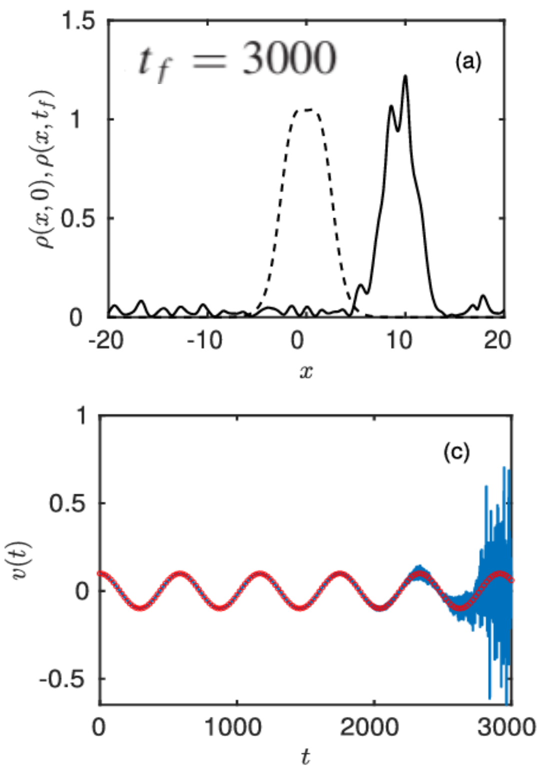
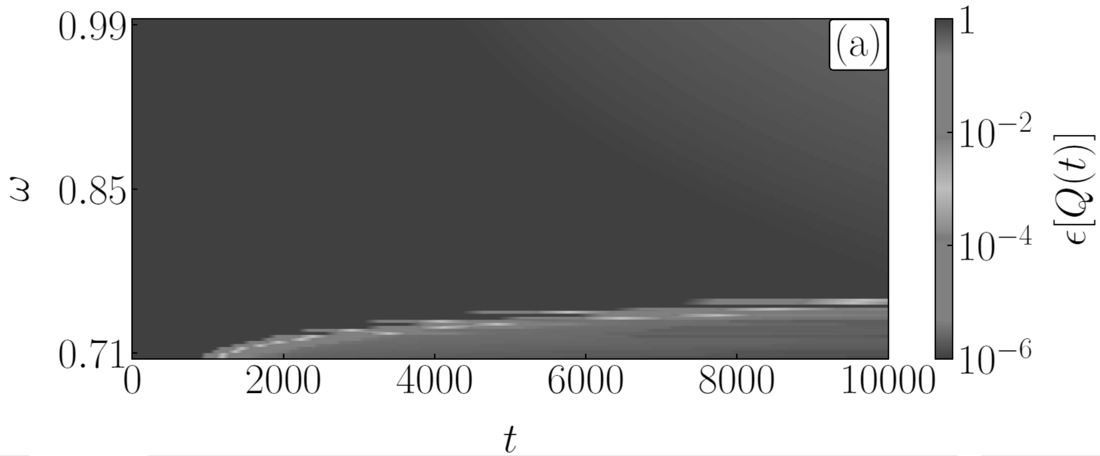
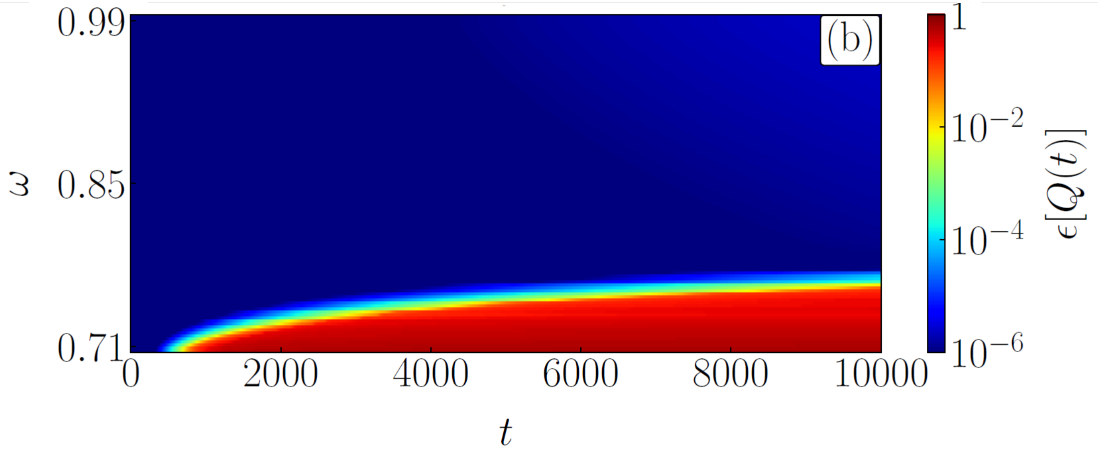
$V_2 = 10^{-4}, \omega = 0.74, q(0) = 0, \dot{q}(0) = 0.1$



(a) $V_2 = 0$ (without potential) and (b) $V_2 = 10^{-4}$, $L = 100$, $q(0) = 0$, $\dot{q}(0) = 0.1$, $p(0) = p(\omega)$

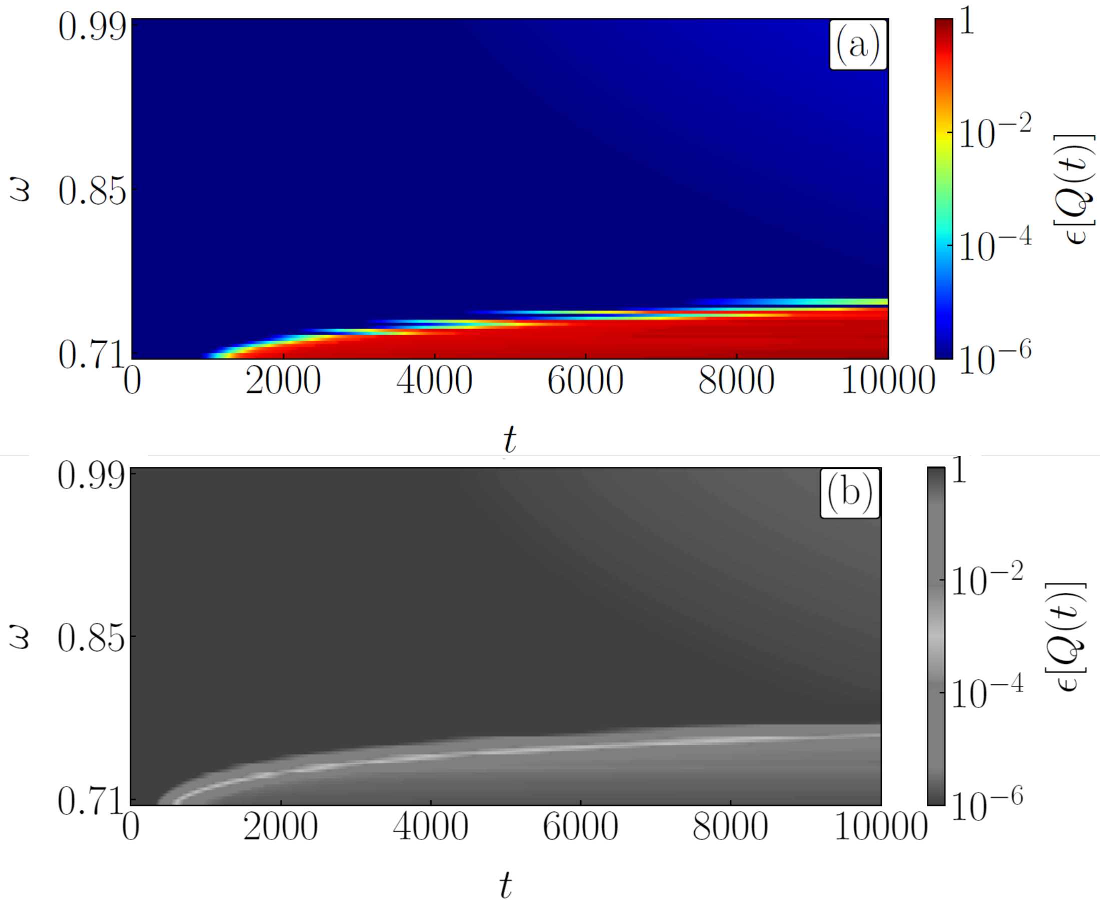




ABS MODEL: HARMONIC POTENTIAL $V(x) = \frac{1}{2}V_2 x^2$

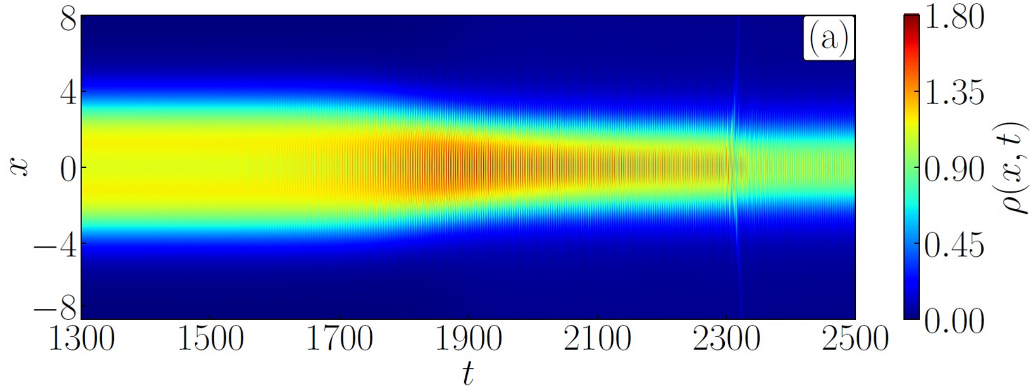
INSTABILITIES PERSIST WHEN ABSORBING BOUNDARY CONDITIONS ARE APPLIED!!

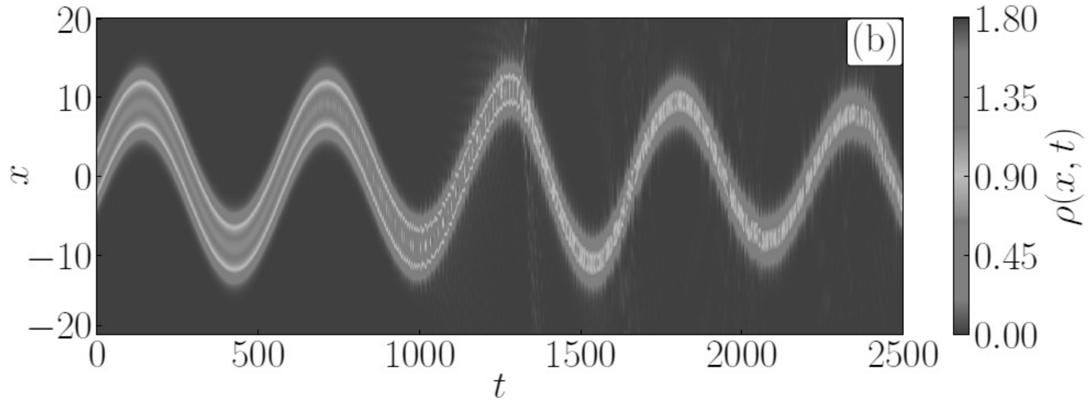
(a) $V_2 = 0$ (without potential) and (b) $V_2 = 10^{-4}$, $L = 100$, $q(0) = 0$, $\dot{q}(0) = 0.1$, $p(0) = p(\omega)$



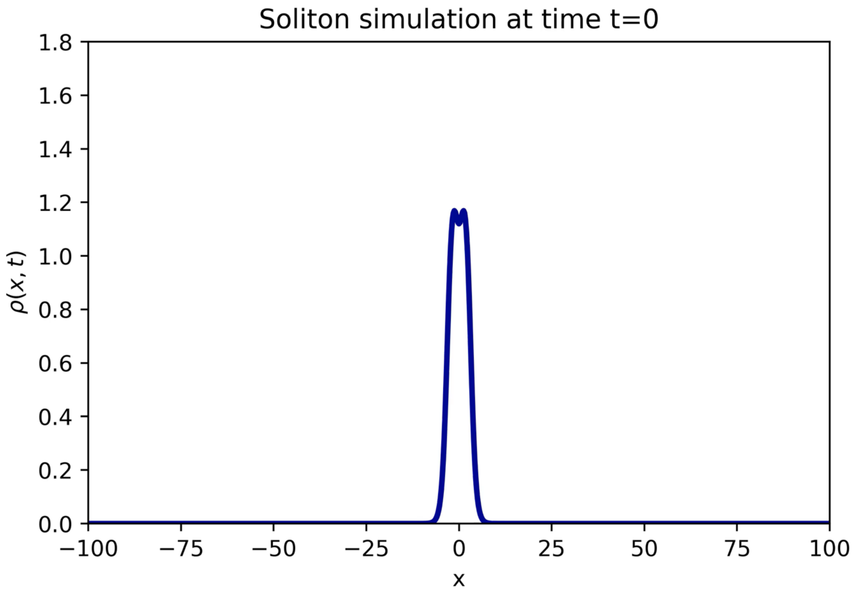
ABS MODEL: HARMONIC POTENTIAL $V(x) = \frac{1}{2}V_2 x^2$

(a) $V_2 = 0$ (without potential) and (b) $V_2 = 10^{-4}$, $L = 100$, $\omega = 0.72$
$q(0) = 0$, $\dot{q}(0) = 0.1$, $p(0) = p(\omega)$



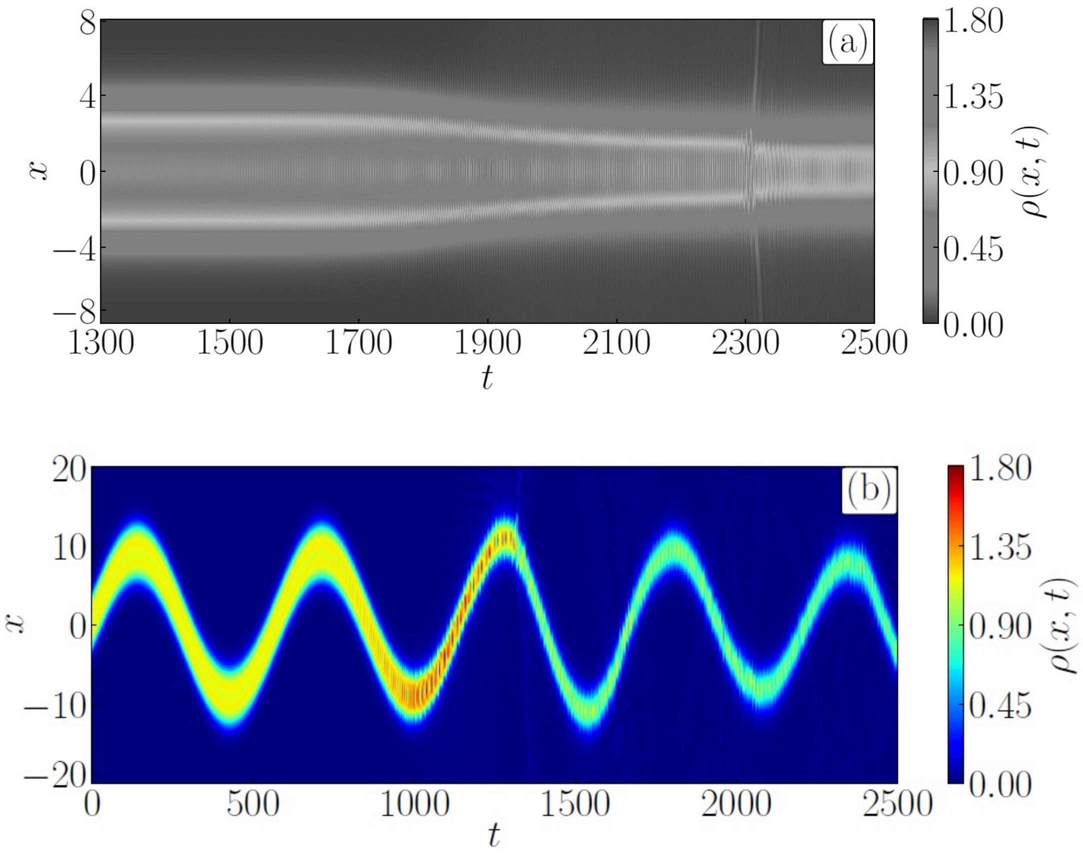


(a) $V_2 = 0$

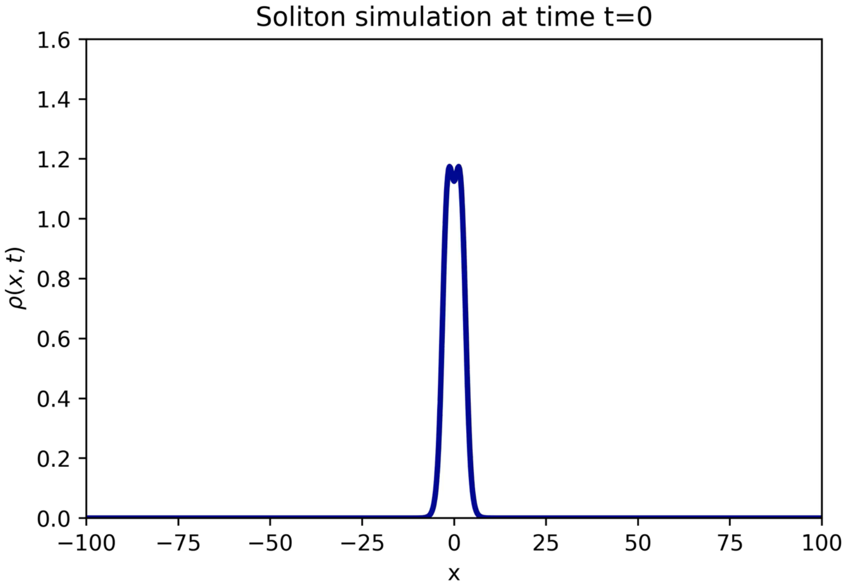


ABS MODEL: HARMONIC POTENTIAL $V(x) = \frac{1}{2} V_2 x^2$

(a) $V_2 = 0$ (without potential) and (b) $V_2 = 10^{-4}$, $L = 100$, $\omega = 0.72$
$q(0) = 0$, $\dot{q}(0) = 0.1$, $p(0) = p(\omega)$



(b) $V_2 = 10^{-4}$



CONCLUSIONS AND OUTLOOK

- New numerical algorithm for NLDE -> convergence for $h \leq 10^{-3}$
- Solitary wave solution may be approximated by an Ansatz with only two Collective Coordinates: q(t) and p(t)
- The instabilities appeared for low frequencies in the GN model are removed by appling absorbing boundary conditions, but q(t) is modified
- The instabilities at low frequencies persist even when absorbing boundary conditions are added to the ABS model with and without potential
- Future works-> Numerical algorithm extended to temporal potentials

Thank you!!

In collaboration with
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